2nd Law of Thermodynamics

$$\int \frac{dQ}{T} + \Delta S_{\text{int.}} = \Delta S$$

Entropy change due to heat transfer

Entropy change due to internal processes – increases as much as possible to reach equilibrium

$$\Delta S_{\substack{\text{(excepting}\\ \text{V dependence)}}} = \begin{cases} \int_{T_1}^{T_2} \frac{mcdT}{T} & \text{T change} \\ mL & \text{phase change} \end{cases}$$

$$S = Nk_B \ln\left(\frac{T^{f/2}V}{\Phi N}\right) \quad \text{ideal gas}$$

$$S = Nk_B \ln\left(\frac{T^{f/2}(V - bN)}{\Phi N}\right) \quad \text{Van der Waals gas}$$

$$S = Nk_B \ln\left(\frac{T^{f/2}V_{micro}}{\Phi N}\right) + \alpha\beta(V - \upsilon) \quad \text{ideal solid}$$

2^{nd} Law of Thermodynamics

It is the 2nd law (in conjunction with the 1st) which mandates any two objects placed in thermal contact must come to equilibrium at the same temperature.

$$Q + W = \Delta E$$

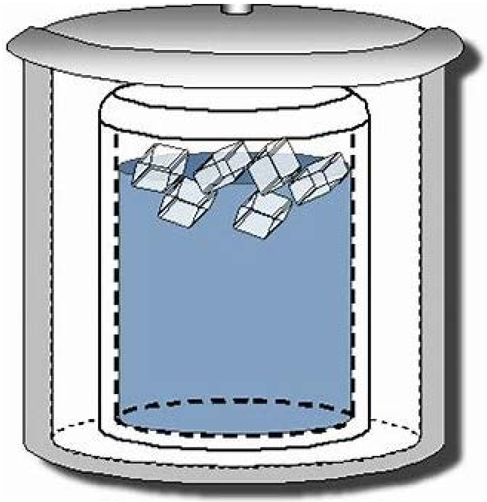
$$0 + 0 = \Delta E$$

This implies that the total E is constant.
So we can write $E_1 + E_2 = E$.

$$\int \frac{dQ}{T} + \Delta S_{\text{int.}} = \Delta S$$

$$0 + (\text{as large as possible}) = \Delta S$$

This implies that the total S is maximized
So we can write $S_1 + S_2 = \text{max}$.



$$S_1(E_1) + S(E_2) = \max$$

$$S_1(E_1) + S_2(E - E_1) = \max$$

$$\frac{d}{dE_1}(S_1 + S_2) = 0$$

any function is max when its derivative is zero

$$\frac{1}{T_1} - \frac{1}{T_2} = 0$$

$$dS = \frac{dE}{T} \rightarrow \frac{dS}{dE} = \frac{1}{T}$$

$$\boxed{T_1 = T_2}$$

2nd Law of Thermodynamics

A coffee cup containing 0.3kg of hot coffee at 90°C. You put in a 20g ice cube at its melting point to cool the coffee. By how many degrees (in Celsius) has your coffee cooled once the ice has melted and equilibrium is reached? Treat the coffee as though it were pure water and neglect energy exchanges with the environment.

$$Q + W = \Delta E$$

$$0 = m_{coffee} c_{water} (T - 90) + m_{ice} L_f + m_{ice} c_{water} (T - 0)$$

$$T = \frac{m_{coffee} c_{water} (90) - m_{ice} L_f}{m_{coffee} c_{water} + m_{ice} c_{water}} = \frac{(0.40)(4.18)(90) - (0.025)(333)}{(0.40)(4.18) + (0.025)(4.18)} = 80^{\circ} C$$

Is the 2nd law satisfied?

$$\int \frac{dQ}{T} + \Delta S_{\text{int.}} = \Delta S_{coffee} + \Delta S_{ice}$$

$$0 + \Delta S_{\text{int.}} = \int_{273+90}^{273+80} \frac{m_{coffee} c_{water} dT}{T} + \frac{m_{ice} L_f}{273} + \int_{273}^{273+80} \frac{m_{ice} c_{water} dT}{T}$$

$$\Delta S_{\text{int.}} = (0.4)(4.18) \ln\left(\frac{273+80}{273+90}\right) + \frac{(0.025)(333)}{273} + (0.025)(4.18) \ln\left(\frac{273+80}{273}\right)$$

$\Delta S_{\text{int.}} = 0.01 \text{J/K}$ Yes, it's satisfied (because its > 0). Awesome!

2^{nd} Law of Thermodynamics

Say our 70kg dude jumps off a 5m cliff, and comes to rest in a 100m^3 pool of water at 30C. What is the temperature increase of the water?

$$Q + W = \Delta E_{person} + \Delta E_{water}$$

$$0 + 0 = mg\Delta y + m_{water} c_{water} \Delta T$$

$$0 = (70)(9.8)(-5) + \left(1000\frac{\text{kg}}{\text{m}^3} \times 100\text{m}^3\right)\left(4.18\times10^3 \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\Delta T$$

$$\Delta T = \frac{(70)(9.8)(5)}{(1000\times100\text{m}^3)(4.18\times10^3)} = 8.2\mu\text{K}$$

Is the 2^{nd} law satisfied?

$$\int \frac{dQ}{T} + \Delta S_{\text{int.}} = \Delta S_{you} + \Delta S_{water}$$

$$0 + \Delta S_{\text{int.}} = 0 + \int_{273+30}^{273+30+8.2\mu\text{K}} \frac{(1000\times100)(4.18\times10^3)dT}{T}$$

$$\Delta S_{\text{int.}} = (1000\times100)(4.18\times10^3)\ln\left(\frac{273+30+8.2\times10^{-6}}{273+30}\right)$$

$$\Delta S_{\text{int.}} = 11.3\,\text{J/K}$$

Yes, it's satisfied (because its > 0).